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Pion-Nucleon Analysis at Low Energy

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1. Motivation

The study of πN interaction is of interest, because

- it is the simplest example of Goldstone-matter interaction,
- advances in χPT will require input,
- there are plenty of data,
- information of the interaction is needed in other fields, e.g., nuclear physics and dark matter searches.

thesis work of Pekko Metsä

2. Recent progress

- VPI-GWU analysis (FA02), http://gwdac.phys.gwu.edu/
  R.A. Arndt et al., Phys. Rev. **C69** (2004) 035213.
  pw's up to 2.1 GeV, fixed-t constraints up to 1 GeV with
  $-0.4\ \mathrm{GeV}^2 \leq t \leq 0.\ \mathrm{GeV}^2$,
  $f^2 = 0.0761 \pm 0.0006$, $a_{\pi^- p} = 0.0856 \pm 0.0010\ \mu^{-1}$
- The analysis of D.V. Bugg, Eur. Phys. J. **C33** (2004) 505.
  $f^2 = 0.0755 - 0.0763 \pm 0.0007$, $a_{\pi^- p} = 0.0850 - 0.0863\ \mu^{-1}$
- Pionic hydrogen analysis
  T.E.O. Ericson et al., Phys. Lett. **B594** (2004) 76.
  $f^2 = 0.0777 \pm 0.0009$, $a_{\pi^- p} = 0.0870 \pm 0.0005\ \mu^{-1}$

3. πN amplitude

$$T_{\pi N} = \bar{u}'[A(\nu,t) + \frac{1}{2}\gamma^\mu (q+q')_\mu B(\nu,t)]u$$

where

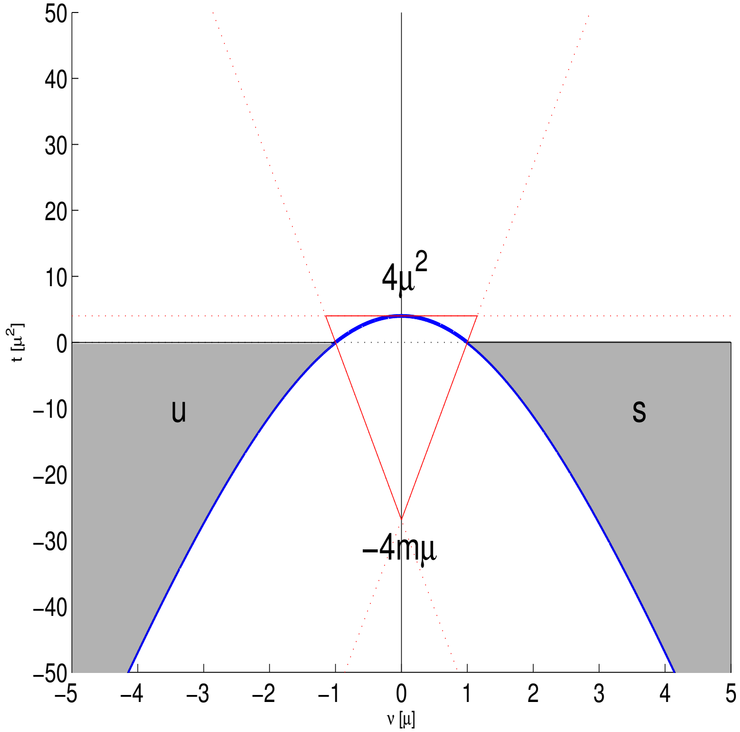
$$\nu = \frac{s-u}{4m} = \omega + \frac{t}{4m},$$

$$C(\nu,t) = A(\nu,t) + \nu/(1-t/4m^2)\, B(\nu,t)$$

Optical theorem: $\mathrm{Im}\, C(\omega, t=0) = k_{\,\mathrm{lab}}\, \sigma$

Isospin: $C^{\pm} = \frac{1}{2}(C_{\pi^- p} \pm C_{\pi^+ p})$

4. Mandelstam diagram



5. Fixed-t analysis

Sensitive quantities like Σ call for a new PWA with fixed-t constraints. In the forward direction it is feasible to solve the dispersion relations directly, but for $t < 0$ it is more practical to use the expansion method.

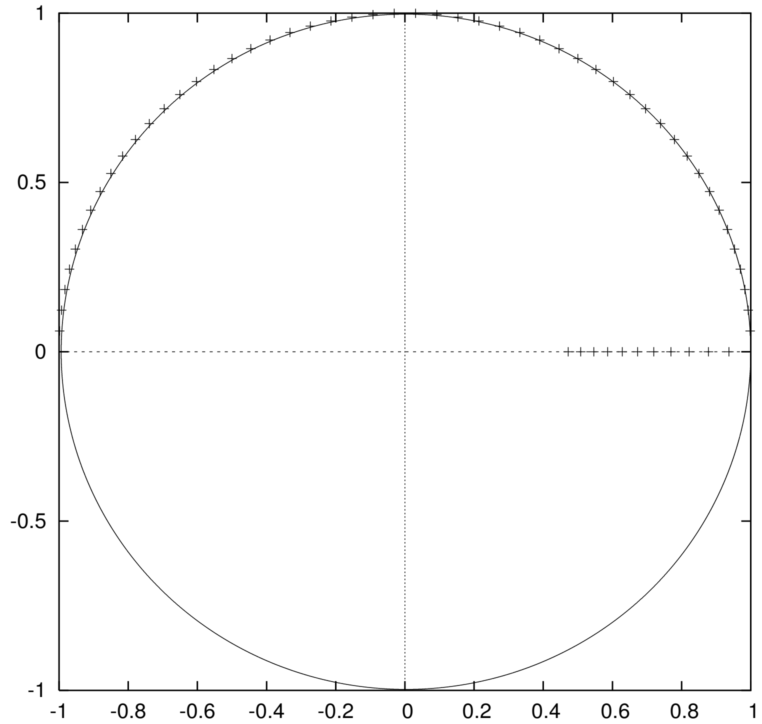
E.g. for the C^+ amplitude:

$$C^+(\nu,t) = C_N^+(\nu,t) + H(Z,t)\sum_{n=0}^{N} c_n^+ Z^n,$$

where H is adjusted to the asymptotic behaviour of the amplitude and

$$Z(\nu^2,t) = \frac{\alpha - \sqrt{\nu_{th}^2 - \nu^2}}{\alpha + \sqrt{\nu_{th}^2 - \nu^2}}.$$

This maps the physical region on the upper semicircle of the unit circle



The convergence and smoothing is taken care by a convergence test function

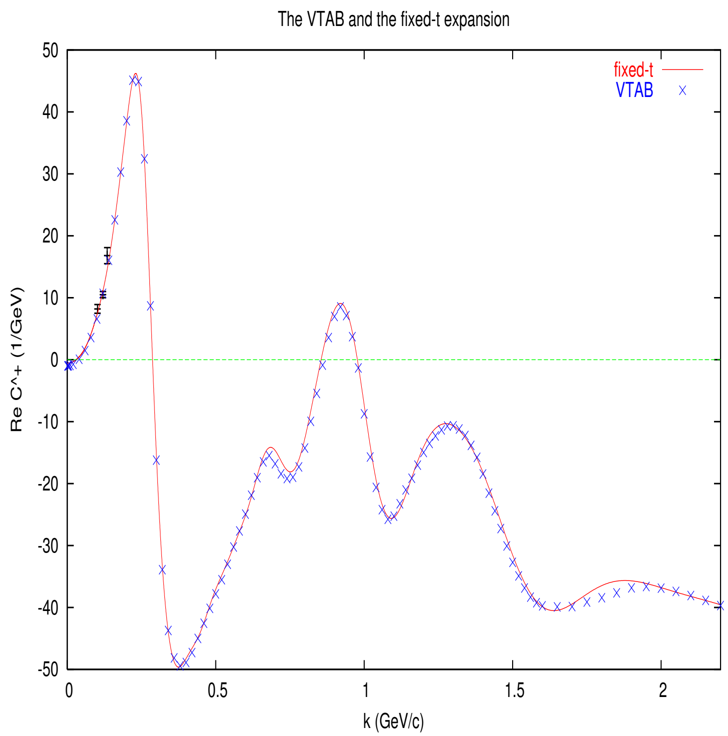
$$\chi_T^2 = \lambda \sum_{n=0}^{N} c_n^2 (n+1)^3,$$

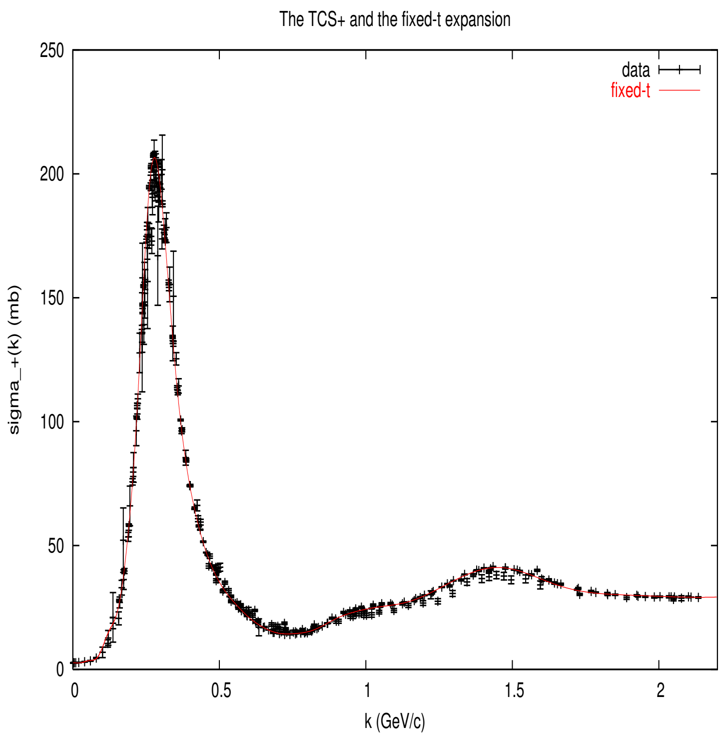
which is added to the χ^2 expression with similar pieces for the C^- and $B^\pm$ amplitudes. The total χ^2 expression is then

$$\chi^2 = \chi^2_{DATA} + \chi^2_{PW} + \chi^2_T (4\,\text{pieces}),$$

where χ^2_{DATA} and χ^2_{PW} refer to the contributions from data and the existing partial wave solution respectively.

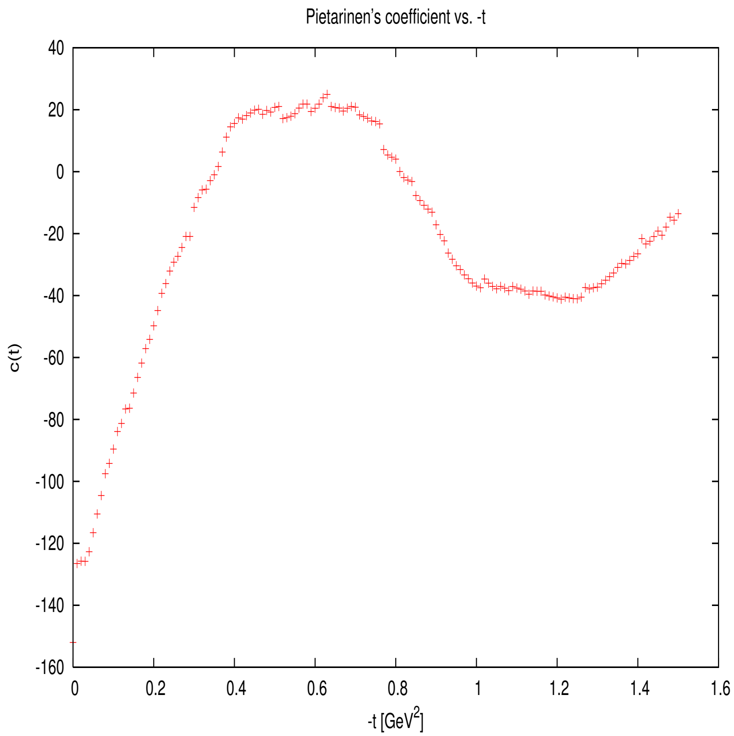
At $t = 0$ with N=40 we have:

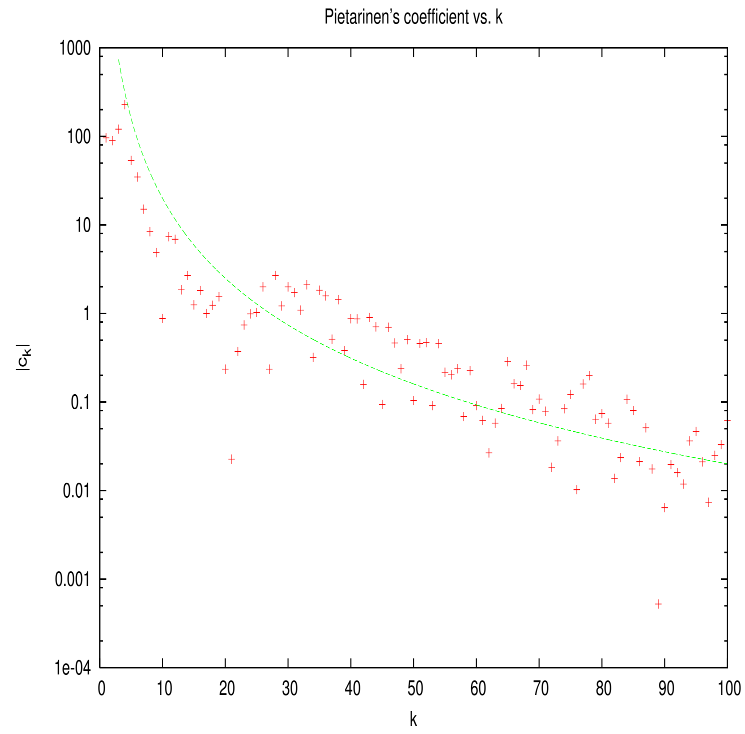




For the real part the input is the 3 experimental low-energy points for $\mathrm{Re}\, D^+(t=0)$ measured by the Karlsruhe-ETH and Karlsruhe-Tübingen groups. The corresponding value for C^+ at the threshold is -0.57 GeV^{-1} giving for the isoscalar s-wave scattering length the value $a^+_{0+} = -0.0051\ \mu^{-1}$.

For the C^+-amplitude we obtain:

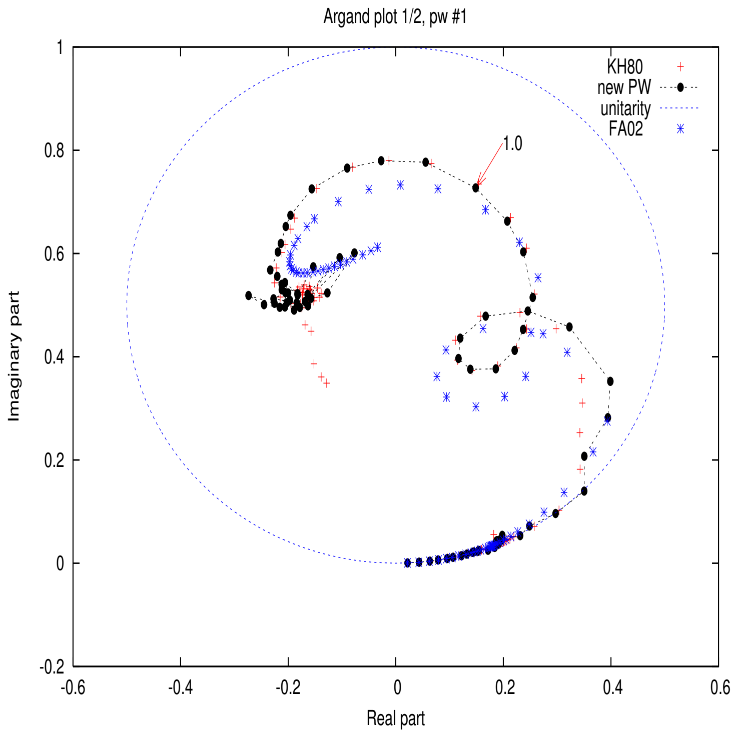


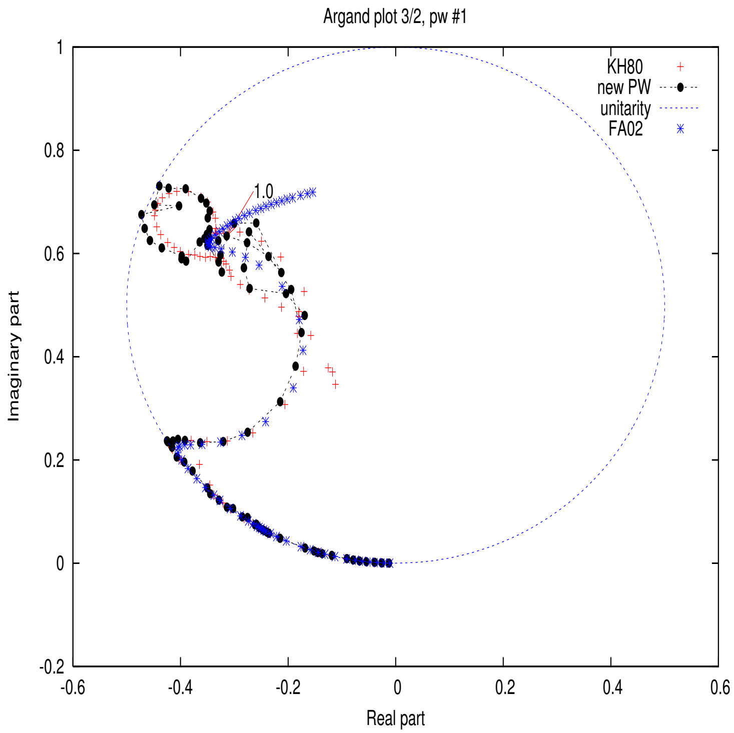


6. Partial waves

For the partial wave analysis the fit includes the experimental data and the fixed-t amplitudes together with the unitarity constraint.

The Argand plots for the s-waves:





7. The plan

- Discrete PSA with input from FDR up to 700 MeV/c

  (in collaboration with Vladimir Abaev, PNPI)

  electromagnetic corrections with Tromborg formalism,

  the P_{33} -splitting included
- Forward dispersion relation analysis (FDR)

  including some reasonable asymptotic behaviour
- Fixed-t analysis for $0 \geq t \geq -1.5$ GeV^2
- New PSA

  with values for Σ, f^2 and the subthreshold coefficients